A note on the t-value and t-related solution concepts

van den Brink, J.R.

Publication date:
1994

45

Faculty of Economics

research memorandum

game theory

Tilburg University

[redacted]

A NOTE ON THE τ-VALUE AND τ-RELATED SOLUTION CONCEPTS

René van den Brink

FEW 652

Communicated by Prof.dr. P.H.M. Ruys

A Note on the τ-Value and τ-Related Solution Concepts*

René van den Brink†

Department of Econometrics
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands

April 1994

*The author would like to thank Peter Borm, Rob Gilles and Gert-Jan Otten for their useful remarks.

†The author is financially supported by the Netherlands Organization for Scientific Research (NWO), grant 450-228-022

Abstract

The τ-*value* is a solution concept for a special class of *cooperative games with transferable utilities* (TU-games). Other solution concepts can be defined and axiomatized in a similar way as the τ-value. We discuss an example of such a τ-related solution concept which is defined for all TU-games.

1 Introduction

A situation in which a set of players N can obtain certain payoffs by cooperation can be described by a *cooperative game with transferable utilities* (or simply a TU-game) being a function $v: 2^N \to \mathbb{R}$ such that $v(\emptyset) = 0$. We denote the class of all TU-games on N by $\mathcal{G}^N$. An *efficient allocation* for $v \in \mathcal{G}^N$ is a vector $x \in \mathbb{R}^N$ such that $\sum_{i\in N} x_i = v(N)$. An efficient *solution concept* for a subclass of games $\mathcal{G} \subset \mathcal{G}^N$ is a function $f: \mathcal{G} \to \mathbb{R}^N$ which assigns to every $v \in \mathcal{G}$ an efficient allocation $f(v) \in \mathbb{R}^N$.

In Tijs (1981) an efficient solution concept for a special class of TU-games is introduced, namely the τ-*value*. First the following two functions on $\mathcal{G}^N$ are defined. The *marginal contribution* is the function $M: \mathcal{G}^N \to \mathbb{R}^N$ given by

$$M_i(v) = v(N) - v(N \setminus \{i\}) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N.$$

The *minimal right* is the function $m: \mathcal{G}^N \to \mathbb{R}^N$ given by

$$m_i(v) = \max_{E\ni i} \left(v(E) - \sum_{j\in E\setminus\{i\}} M_j(v) \right) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N.$$

A game $v \in \mathcal{G}^N$ is called *quasi-balanced* if the following two conditions are satisfied

$$m(v) \leq M(v) \text{ and } \sum_{i\in N} m_i(v) \leq v(N) \leq \sum_{i\in N} M_i(v). \tag{1}$$

The class of all quasi-balanced games on N is denoted by Q^N. If $v \in Q^N$ then the allocations $m(v)$ and $M(v)$, respectively, can be seen as lower and upper bounds for the distribution of the payoffs over the players in N. The τ-value is the function $\tau: Q^N \to \mathbb{R}^N$ which assigns to every quasi-balanced game the unique efficient allocation on the line segment between $m(v)$ and $M(v)$, i.e., for every $v \in Q^N$ it holds that

$\tau(v) = m(v) + \alpha_v(M(v) - m(v))$,

where $\alpha_v = \begin{cases} \frac{v(N) - \sum_{i \in N} m_i(v)}{\sum_{i \in N} M_i(v) - \sum_{i \in N} m_i(v)} & \text{if } m(v) \neq M(v) \\ 0 & \text{else.} \end{cases}$

For every $v \in \mathcal{G}^N$, $k \in \mathbb{R}_+$ and $c \in \mathbb{R}^N$ the game $(kv + c)$ is given by $(kv + c)(E) = kv(E) + \sum_{i \in E} c_i$ for all $E \subset N$. In Tijs (1987) the following axiomatization of the τ-value is presented.

Theorem 1.1 (Tijs (1987)) *The efficient solution concept $f: Q^N \to \mathbb{R}^N$ is equal to the τ-value if and only if the following two conditions are satisfied:*

(i) *for every $v \in Q^N$ it holds that $f(v) = m(v) + f(v - m(v))$ (minimal right property);*

(ii) *for every $v \in Q_0^N := \{v \in Q^N \mid m(v) = 0\}$ the vector $f(v) \in \mathbb{R}^N$ is proportional to the vector $M(v) \in \mathbb{R}^N$. (restricted proportionality property).*

2 A τ-related solution concept for all TU-games

A disadvantage of the τ-value is that it does not exist for games that are not quasi-balanced. This is because there are games for which the marginal contribution and minimal right are inadequate as upper and lower bounds for the distribution of payoffs. But for such games other bounds can be appropriate. If we take two functions $L: \mathcal{G}^N \to \mathbb{R}^N$ and $U: \mathcal{G}^N \to \mathbb{R}^N$ such that for game v the conditions stated under (1) are satisfied with m and M replaced by L and U, respectively, then the values $L(v)$ and $U(v)$ can be seen as lower and upper bounds for the distribution of payoffs in game v. Let $\mathcal{G}^N(L, U) \subset \mathcal{G}^N$ denote the class of games for which the conditions under (1) are satisfied in terms of L and U. Then we define $t^{\{(L,U\}}: \mathcal{G}^N(L, U) \to \mathbb{R}^N$ as the function which assigns to every $v \in \mathcal{G}^N(L, U)$ the unique efficient allocation on the line segment between $L(v)$ and $U(v)$. (Thus $\tau = t^{\{m,M\}}$.) Moreover, if L and U satisfy the S-equivalence property* then $t^{\{L,U\}}$ can be axiomatized similarly as the τ-value by

*The function $f: \mathcal{G} \to \mathbb{R}^N$, satisfies the *S-equivalence* property on $\mathcal{G} \subset \mathcal{G}^N$ if for every $v, w \in \mathcal{G}$, $k \in \mathbb{R}_+$ and $c \in \mathbb{R}^N$ such that $w = kv + c$, it holds that $f(w) = kf(v) + c$.

replacing m and M in Theorem 1.1 by L and U, respectively. (The proof of this result is similar to the proof of Theorem 1.1 as given in Tijs (1987)[†].) We refer to solution concepts that can be obtained in this way as *τ-related solution concepts*.

Most τ-related solution concepts are only defined for specific subclasses of games that satisfy the conditions stated under (1). However, next we present a τ-related solution concept that is defined for all TU-games. As lower and upper bounds for the final payoff of a player we take the minimal and maximal contribution of that player to any coalition. Thus we define $\widehat{L}: \mathcal{G}^N \to \mathbb{R}^N$ and $\widehat{U}: \mathcal{G}^N \to \mathbb{R}^N$ by

$$\widehat{L}_i(v) = \min_{E \ni i}(v(E) - v(E \setminus \{i\})) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N,$$

and

$$\widehat{U}_i(v) = \max_{E \ni i}\,(v(E) - v(E \setminus \{i\})) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N.$$

The lower bound $\widehat{L}$ has been considered in Kikuta (1980), and the upper bound $\widehat{U}$ in Milnor (1952). This pair of functions forms reasonable upper and lower bounds for any game on N.

Theorem 2.1 *For every $v \in \mathcal{G}^N$ it holds that*

$$\widehat{L}(v) \leq \widehat{U}(v) \text{ and } \sum_{i \in N} \widehat{L}_i(v) \leq v(N) \leq \sum_{i \in N} \widehat{U}_i(v).$$

PROOF

Let $v \in \mathcal{G}^N$ and let $\pi: N \to N$ be a permutation on N. Further, let $P(i, \pi) := \{j \in N \mid \pi(j) < \pi(i)\}$, being the collection of players that precede player i in permutation π. Then we define $m^\pi(v) \in \mathbb{R}^N$ by

$$m_i^\pi(v) = v(P(i, \pi) \cup \{i\}) - v(P(i, \pi)) \text{ for all } i \in N.$$

It is easy to verify that (i) $\widehat{L}(v) \leq m^\pi(v)$, (ii) $\widehat{U}(v) \geq m_i^\pi(v)$, and (iii) $\sum_{i \in N} m_i^\pi(v) = v(N)$.

[†]In that proof only twice use is made of the specific formula's of $m(v)$ and $M(v)$. This is to show that for every $v \in \mathcal{G}^N$, $m(v - m(v))$ is the null vector, and to show that M satisfies the S-equivalence property.

From this it follows that $\widehat{L}(v) \leq \widehat{U}(v)$ and $\sum_{i\in N} \widehat{L}_i(v) \leq v(N) \leq \sum_{i\in N} \widehat{U}_i(v)$.

$\square$

Next we define the function $\widehat{f}: \mathcal{G}^N \to \mathbb{R}^N$ which assigns to every game $v \in \mathcal{G}^N$ the unique efficient vector on the line segment between $\widehat{L}$ and $\widehat{U}$. Since $\widehat{U}$ and $\widehat{L}$ satisfy the S-equivalence property the function $\widehat{f}$ is a τ-related solution concept, and thus can be axiomatized as in Theorem 1.1 by replacing m and M by $\widehat{L}$ and $\widehat{U}$, respectively.

Similarly as shown in Tijs (1981) for the τ-value, it can be shown that $\widehat{f}$ is continuous, and satisfies the anonimity and zero player properties[‡]. Besides these properties it turns out that the value that $\widehat{f}$ assigns to a TU-game is equal to the value that it assigns to its *dual* game.

Theorem 2.2 *Let $v \in \mathcal{G}^N$ and let $v^* \in \mathcal{G}^N$ be the dual game of v, i.e.,*

$$v^*(E) = v(N) - v(N \setminus E) \text{ for all } E \subset N.$$

Then $\widehat{f}(v) = \widehat{f}(v^*)$.

Proof

Let $v \in \mathcal{G}^N$ and let v^* be the dual game of v. For every $i \in N$ it then holds that

$$\widehat{L}_i(v^*) = \min_{E \ni i} \left(v^*(E) - v^*(E \setminus \{i\})\right) = \min_{E \ni i} \left(v(N) - v(N \setminus E) - v(N) + v(N \setminus (E \setminus \{i\}))\right)$$

$$= \min_{E \ni i} \left(v((N \setminus E) \cup \{i\}) - v(N \setminus E)\right) = \min_{F \ni i} \left(v(F) - v(F \setminus \{i\})\right) = \widehat{L}_i(v).$$

Similarly we can prove that $\widehat{U}(v^*) = \widehat{U}(v)$, and thus $\widehat{f}(v^*) = \widehat{f}(v)$.

$\square$

To conclude this section we show that for *convex* games the function $\widehat{f}$ coincides with the τ-value.

[‡]The function $f: \mathcal{G} \to \mathbb{R}^N$ satisfies the *anonimity* property on $\mathcal{G} \subset \mathcal{G}^N$ if for every permutation $\pi: N \to N$ it holds that $f_i(v) = f_{\pi(i)}(v)(\pi v)$ for all $i \in N$ and $v \in \mathcal{G}$, where $\pi v: 2^N \to \mathbb{R}$ is given by: $\pi v(E) = v(\bigcup_{i \in E} \pi(i))$ for all $E \subset N$.

The f function $f: \mathcal{G} \to \mathbb{R}^N$ satisfies the *zero player* property on $\mathcal{G} \subset \mathcal{G}^N$ if for every $v \in \mathcal{G}$ and every player $i \in N$ with $v(E) = v(E \setminus \{i\})$ for all $E \subset N$, it holds that $f_i(v) = 0$.

Proposition 2.3 *Let $v \in \mathcal{G}^N$ be convex, i.e., $v(E \cup F) + v(E \cap F) \geq v(E) + v(F)$ for all $E, F \subset N$. Then $\widehat{f}(v) = \tau(v)$.*

PROOF
Let $v \in \mathcal{G}^N$ be convex, and $i \in N$ and $F \supset E \ni i$.
By convexity of v it holds that

$$v(F) + v(E \setminus \{i\}) = v(E \cup (F \setminus \{i\})) + v(E \cap (F \setminus \{i\})) \geq v(E) + v(F \setminus \{i\}).$$

Thus $v(F) - v(F \setminus \{i\}) \geq v(E) - v(E \setminus \{i\})$ for all $F \supset E \ni i$.
From this it follows that

(i) $\widehat{U}_i(v) = \max_{E \ni i} (v(E) - v(E \setminus \{i\})) = v(N) - v(N \setminus \{i\}) = M_i(v)$, and

(ii) $\widehat{L}_i(v) = \min_{E \ni i} (v(E) - v(E \setminus \{i\})) = v(\{i\})$.

In Driessen and Tijs (1985) it is shown that for a convex game v it holds that $m_i(v) = v(\{i\})$ for all $i \in N$.
Thus $\widehat{f}(v) = \tau(v)$.

□

3 Other τ-related solution concepts

We conclude this note by mentioning some examples of other τ-related solution concepts. In van den Brink (1989) a reasoning is given why for some games the marginal contribution and minimal right can be seen as lower and upper bounds, respectively (so their roles are reversed compared to their roles in determining the τ-value). Consider, for example, the simple majority game v on $N = \{1, 2, 3\}$ which assigns the value 1 to all coalitions that contain two or three players, and the value zero to all other coalitions. In this game the 'grand coalition' N is a 'winning' coalition. If an individual player leaves the grand coalition then the remaining players still form a winning coalition. Thus the marginal contribution of every player equals zero. The minimal right of every player then equals one. Thus the marginal contribution and minimal right, respectively, can be seen as lower and upper bounds for the distribution of payoffs. The function $t^{\{M,m\}}: \mathcal{G}^N(M,m) \to \mathbb{R}^N$ that assigns to every $v \in \mathcal{G}^N(M,m)$ the unique efficient vector on the line segment between M and m, is a τ-related solution concept.

Other possible lower and upper bounds are the functions $\overline{L}: \mathcal{G}^N \to \mathbb{R}^N$ and $\overline{U}: \mathcal{G}^N \to \mathbb{R}^N$ given by

$$\overline{L}_i(v) = v(\{i\}) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N,$$

and

$$\overline{U}_i(v) = v(N) - \sum_{j \in N \setminus \{i\}} \overline{L}_j(v) = v(N) - \sum_{j \in N \setminus \{i\}} v(\{j\}) \text{ for all } i \in N \text{ and } v \in \mathcal{G}^N.$$

The function $t^{\{\overline{L},\overline{U}\}}: \mathcal{G}^N(\overline{L},\overline{U}) \to \mathbb{R}^N$ which assigns to every $v \in \mathcal{G}^N(\overline{L},\overline{U})$ the unique efficient vector on the line segment between $\overline{L}(v)$ and $\overline{U}(v)$, is equal to the *CIS-value* as considered in Driessen and Funaki (1991), i.e., $t_i^{\{\overline{L},\overline{U}\}}(v) = v(\{i\}) + \frac{v(N) - \sum_{j \in N} v(\{j\})}{\#N}$ for all $i \in N$ and $v \in \mathcal{G}^N(\overline{L},\overline{U})$.

The following theorem characterizes the class $\mathcal{G}^N(\overline{L},\overline{U})$. It also shows that for many games, in particular for superadditive games, the functions $t^{\{\overline{L},\overline{U}\}}$ and $t^{\{M,m\}}$ are related through duality.

Theorem 3.1 *Let $v \in \mathcal{G}^N$. Then $v \in \mathcal{G}^N(\overline{L},\overline{U})$ if and only if $v(N) \geq \sum_{i \in N} v(\{i\})$. Moreover, if $v(E) \geq \sum_{i \in E} v(\{i\})$ for all $E \subset N$, then $t^{\{\overline{L},\overline{U}\}}(v) = t^{\{M,m\}}(v^*)$, where v^* is the dual game of v.*

PROOF

Let $v \in \mathcal{G}^N$. It is easy to verify that $v \in \mathcal{G}^N(\overline{L},\overline{U})$ if and only if $v(N) \geq \sum_{i \in N} v(\{i\})$. Suppose that $v(E) \geq \sum_{i \in N} v(\{i\})$ for all $E \subset N$. We show that the lower and upper bounds of the CIS-value $t^{\{\overline{L},\overline{U}\}}(v)$ coincide with the corresponding bounds of $t^{\{M,m\}}(v^*)$. For every $i \in N$ we can derive that

$$M_i(v^*) = v^*(N) - v^*(N \setminus \{i\}) = v(N) - v(\emptyset) - (v(N) - v(\{i\})) = v(\{i\}) = \overline{L}_i(v)$$

and

$$m_i(v^*) = \max_{E \ni i} \left(v^*(E) - \sum_{j \in E \setminus \{i\}} M_j(v^*) \right) = \max_{E \ni i} \left(v(N) - v(N \setminus E) - \sum_{j \in E \setminus \{i\}} v(\{j\}) \right)$$

$$= v(N) - \min_{E \ni i} \left(v(N \setminus E) + \sum_{j \in E \setminus \{i\}} v(\{j\}) \right) = v(N) - \min_{F \not\ni i} \left(v(F) + \sum_{j \in N \setminus (F \cup \{i\})} v(\{j\}) \right)$$

Since $v(E) \geq \sum_{i \in E} v(\{i\})$ for all $E \subset N$ it holds that the minimum in the last expression is obtained for $F = \emptyset$. Thus

$$m_i(v^*) = v(N) - \sum_{j \in N \setminus \{i\}} v(\{j\}) = \overline{U}_i(v).$$

□

More τ-related solution concepts can be found in the survey paper by Tijs and Otten (1993).

References

BRINK, R. VAN DEN (1989), *Analysis of Social Positions of Agents in Hierarchically Structured Organizations,* Master Thesis, Tilburg University, Tilburg.

DRIESSEN, T.S.H., AND Y. FUNAKI (1991), "Coincidence of and Collinearity between Game Theoretic Solutions", *OR Spektrum*, 13, 15-30.

DRIESSEN, T.S.H., AND S.H. TIJS (1985), "The τ-value, the Core and Semiconvex Games", *International Journal of Game Theory*, 14, 229-248.

KIKUTA, K. (1980), "The Smallest Incremental Contribution of each Player and the Core in an n-Person Characteristic Function Game", *Math. Japonica, 24,* 495-506.

MILNOR, J.W. (1952), *Reasonable Outcomes for n-Person Games*, Research Memorandum RM-916, The RAND Corporation, Santa Monica.

TIJS, S.H. (1981), "Bounds for the Core and the τ-value", in O.Moeschlin and D. Pallaschke (eds.), *Game Theory and Mathematical Economics,* North-Holland Publishing Company, 123-132, Amsterdam.

TIJS, S.H. (1987), "An Axiomatization of the τ-value", *Mathematical Social Sciences*, 13, 177-181.

TIJS, S.H., AND G.J. OTTEN (1993), *Compromise values in Cooperative Game Theory*, FEW Research Memorandum 615, Tilburg University, Tilburg.

IN 1993 REEDS VERSCHENEN

588 Rob de Groof and Martin van Tuijl
The Twin-Debt Problem in an Interdependent World
Communicated by Prof.dr. Th. van de Klundert

589 Harry H. Tigelaar
A useful fourth moment matrix of a random vector
Communicated by Prof.dr. B.B. van der Genugten

590 Niels G. Noorderhaven
Trust and transactions; transaction cost analysis with a differential behavioral assumption
Communicated by Prof.dr. S.W. Douma

591 Henk Roest and Kitty Koelemeijer
Framing perceived service quality and related constructs A multilevel approach
Communicated by Prof.dr. Th.M.M. Verhallen

592 Jacob C. Engwerda
The Square Indefinite LQ-Problem: Existence of a Unique Solution
Communicated by Prof.dr. J. Schumacher

593 Jacob C. Engwerda
Output Deadbeat Control of Discrete-Time Multivariable Systems
Communicated by Prof.dr. J. Schumacher

594 Chris Veld and Adri Verboven
An Empirical Analysis of Warrant Prices versus Long Term Call Option Prices
Communicated by Prof.dr. P.W. Moerland

595 A.A. Jeunink en M.R. Kabir
De relatie tussen aandeelhoudersstructuur en beschermingsconstructies
Communicated by Prof.dr. P.W. Moerland

596 M.J. Coster and W.H. Haemers
Quasi-symmetric designs related to the triangular graph
Communicated by Prof.dr. M.H.C. Paardekooper

597 Noud Gruijters
De liberalisering van het internationale kapitaalverkeer in historisch-institutioneel perspectief
Communicated by Dr. H.G. van Gemert

598 John Görtzen en Remco Zwetheul
Weekend-effect en dag-van-de-week-effect op de Amsterdamse effectenbeurs?
Communicated by Prof.dr. P.W. Moerland

599 Philip Hans Franses and H. Peter Boswijk
Temporal aggregration in a periodically integrated autoregressive process
Communicated by Prof.dr. Th.E. Nijman

600 René Peeters
On the p-ranks of Latin Square Graphs
Communicated by Prof.dr. M.H.C. Paardekooper

601 Peter E.M. Borm, Ricardo Cao, Ignacio García-Jurado
Maximum Likelihood Equilibria of Random Games
Communicated by Prof.dr. B.B. van der Genugten

602 Prof.dr. Robert Bannink
Size and timing of profits for insurance companies. Cost assignment for products with multiple deliveries.
Communicated by Prof.dr. W. van Hulst

603 M.J. Coster
An Algorithm on Addition Chains with Restricted Memory
Communicated by Prof.dr. M.H.C. Paardekooper

604 Ton Geerts
Coordinate-free interpretations of the optimal costs for LQ-problems subject to implicit systems
Communicated by Prof.dr. J.M. Schumacher

605 B.B. van der Genugten
Beat the Dealer in Holland Casino's Black Jack
Communicated by Dr. P.E.M. Borm

606 Gert Nieuwenhuis
Uniform Limit Theorems for Marked Point Processes
Communicated by Dr. M.R. Jaïbi

607 Dr. G.P.L. van Roij
Effectisering op internationale financiële markten en enkele gevolgen voor banken
Communicated by Prof.dr. J. Sijben

608 R.A.M.G. Joosten, A.J.J. Talman
A simplicial variable dimension restart algorithm to find economic equilibria on the unit simplex using n(n+1) rays
Communicated by Prof.Dr. P.H.M. Ruys

609 Dr. A.J.W. van de Gevel
The Elimination of Technical Barriers to Trade in the European Community
Communicated by Prof.dr. H. Huizinga

610 Dr. A.J.W. van de Gevel
Effective Protection: a Survey
Communicated by Prof.dr. H. Huizinga

611 Jan van der Leeuw
First order conditions for the maximum likelihood estimation of an exact ARMA model
Communicated by Prof.dr. B.B. van der Genugten

612 Tom P. Faith
Bertrand-Edgeworth Competition with Sequential Capacity Choice
Communicated by Prof.Dr. S.W. Douma

613 Ton Geerts
The algebraic Riccati equation and singular optimal control: The discrete-time case
Communicated by Prof.dr. J.M. Schumacher

614 Ton Geerts
Output consistency and weak output consistency for continuous-time implicit systems
Communicated by Prof.dr. J.M. Schumacher

615 Stef Tijs, Gert-Jan Otten
Compromise Values in Cooperative Game Theory
Communicated by Dr. P.E.M. Borm

616 Dr. Pieter J.F.G. Meulendijks and Prof.Dr. Dick B.J. Schouten
Exchange Rates and the European Business Cycle: an application of a 'quasi-empirical' two-country model
Communicated by Prof.Dr. A.H.J.J. Kolnaar

617 Niels G. Noorderhaven
The argumentational texture of transaction cost economics
Communicated by Prof.Dr. S.W. Douma

618 Dr. M.R. Jaïbi
Frequent Sampling in Discrete Choice
Communicated by Dr. M.H. ten Raa

619 Dr. M.R. Jaïbi
A Qualification of the Dependence in the Generalized Extreme Value Choice Model
Communicated by Dr. M.H. ten Raa

620 J.J.A. Moors, V.M.J. Coenen, R.M.J. Heuts
Limiting distributions of moment- and quantile-based measures for skewness and kurtosis
Communicated by Prof.Dr. B.B. van der Genugten

621 Job de Haan, Jos Benders, David Bennett
Symbiotic approaches to work and technology
Communicated by Prof.dr. S.W. Douma

622 René Peeters
Orthogonal representations over finite fields and the chromatic number of graphs
Communicated by Dr.ir. W.H. Haemers

623 W.H. Haemers, E. Spence
Graphs Cospectral with Distance-Regular Graphs
Communicated by Prof.dr. M.H.C. Paardekooper

624 Bas van Aarle
The target zone model and its applicability to the recent EMS crisis
Communicated by Prof.dr. H. Huizinga

625 René Peeters
Strongly regular graphs that are locally a disjoint union of hexagons
Communicated by Dr.ir. W.H. Haemers

626 René Peeters
Uniqueness of strongly regular graphs having minimal p-rank
Communicated by Dr.ir. W.H. Haemers

627 Freek Aertsen, Jos Benders
Tricks and Trucks: Ten years of organizational renewal at DAF?
Communicated by Prof.dr. S.W. Douma

628 Jan de Klein, Jacques Roemen
Optimal Delivery Strategies for Heterogeneous Groups of Porkers
Communicated by Prof.dr. F.A. van der Duyn Schouten

629 Imma Curiel, Herbert Hamers, Jos Potters, Stef Tijs
The equal gain splitting rule for sequencing situations and the general nucleolus
Communicated by Dr. P.E.M. Borm

630 A.L. Hempenius
Een statische theorie van de keuze van bankrekening
Communicated by Prof.Dr.Ir. A. Kapteyn

631 Cok Vrooman, Piet van Wijngaarden, Frans van den Heuvel
Prevention in Social Security: Theory and Policy Consequences
Communicated by Prof.Dr. A. Kolnaar

IN 1994 REEDS VERSCHENEN

632 B.B. van der Genugten
Identification, estimating and testing in the restricted linear model
Communicated by Dr. A.H.O. van Soest

633 George W.J. Hendrikse
Screening, Competition and (De)Centralization
Communicated by Prof.dr. S.W. Douma

634 A.J.T.M. Weeren, J.M. Schumacher, and J.C. Engwerda
Asymptotic Analysis of Nash Equilibria in Nonzero-sum Linear-Quadratic Differential Games. The Two-Player case
Communicated by Prof.dr. S.H. Tijs

635 M.J. Coster
Quadratic forms in Design Theory
Communicated by Dr.ir. W.H. Haemers

636 Drs. Erwin van der Krabben, Prof.dr. Jan G. Lambooy
An institutional economic approach to land and property markets - urban dynamics and institutional change
Communicated by Dr. F.W.M. Boekema

637 Bas van Aarle
Currency substitution and currency controls: the Polish experience of 1990
Communicated by Prof.dr. H. Huizinga

638 J. Bell
Joint Ventures en Ondernemerschap: Interpreneurship
Communicated by Prof.dr. S.W. Douma

639 Frans de Roon and Chris Veld
Put-call parities and the value of early exercise for put options on a performance index
Communicated by Prof.dr. Th.E. Nijman

640 Willem J.H. Van Groenendaal
Assessing demand when introducing a new fuel: natural gas on Java
Communicated by Prof.dr. J.P.C. Kleijnen

641 Henk van Gemert & Noud Gruijters
Patterns of Financial Change in the OECD area
Communicated by Prof.dr. J.J Sijben

642 Drs. M.R.R. van Bremen, Drs. T.A. Marra en Drs. A.H.F. Verboven
Aardappelen, varkens en de termijnhandel: de reële optietheorie toegepast
Communicated by Prof.dr. P.W. Moerland

643 W.J.H. Van Groenendaal en F. De Gram
The generalization of netback value calculations for the determination of industrial demand for natural gas
Communicated by Prof.dr. J.P.C. Kleijnen

644 Karen Aardal, Yves Pochet and Laurence A. Wolsey
Capacitated Facility Location: Valid Inequalities and Facets
Communicated by Dr.ir. W.H. Haemers

645 Jan J.G. Lemmen
An Introduction to the Diamond-Dybvig Model (1983)
Communicated by Dr. S. Eijffinger

646 Hans J. Gremmen and Eva van Deurzen-Mankova
Reconsidering the Future of Eastern Europe: The Case of Czecho-Slovakia
Communicated by Prof.dr. H.P. Huizinga

647 H.M. Webers
Non-uniformities in spatial location models
Communicated by Prof.dr. A.J.J. Talman

648 Bas van Aarle
Social welfare effects of a common currency
Communicated by Prof.dr. H. Huizinga

649 Laurence A.G.M. van Lent
De winst is absoluut belangrijk!
Communicated by Prof.drs. G.G.M. Bak

650 Bert Hamminga
Jager over de theorie van de internationale handel
Communicated by Prof.dr. H. Huizinga

651 J.Ch. Caanen and E.N. Kertzman
A comparison of two methods of inflation adjustment
Communicated by Prof.dr. J.A.G. van der Geld